A simpler and faster method for SVM implementation

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Abstract

Space Vector Modulation is perhaps the common technique mostly applied to drive three-phase voltage-source inverters. During every switching period it calculates three duty cycles in order to generate a suitable pulse sequence. This paper presents a new, faster and, most important, simpler method to compute these time values without using either trigonometric functions or even Clarke or Park transformations. The result is a light-weight algorithm easier to implement in small digital signal processors or microcontrollers. The relationship between SVM and PWM is also explained.

Introduction

Space Vector Modulation (SVM) was originally developed by Van der Broeck et al. [1] as a vector approach to pulse width modulation (PWM) for three-phase voltage source inverters (VSI). Compared with the former three-phase sinusoidal modulation method (SPWM) it has the advantages of lower current harmonics and a possible higher modulation index [1]. This technique has a wide linear modulation range with no need of distorted modulation and it also guarantees that only one switch changes at any time.

During two decades, SVM has been considered a reference for most analysis, comparisons and physical verifications [2]–[6]. Recently, some research efforts have been directed to develop multilevel inverters and matrix converters based on SVM method [7], [8]. However, recent studies have been more critical with SVM and several solutions in time domain are defended as simpler and/or faster [9], [10].

This paper improves the equations used to compute SVM duty cycles for two-level inverters and reveals its relationship with carrier-based PWM methods.

Classical SVM

The structure of a typical three-phase voltage source power inverter is shown in Fig. 1. The relationship between switching variables ($S_u$, $S_v$, $S_w$) and line-to-line voltages ($V_{uv}$, $V_{vw}$, $V_{wu}$) is given by (1), where $V_{dc}$ is the bus voltage and $S_k$ is '1' or '0' when the upper or lower transistor of phase $k$ is on, respectively. Choosing a neutral point 'N' that satisfies (2) and equating (1) it yields (3).
Fig. 1: Hex bridge, with the upper switches used to define switching states.

\[
\begin{bmatrix}
V_{UV} \\
V_{VW} \\
V_{WU}
\end{bmatrix}
= \begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & -1 \\
-1 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
S_u \\
S_v \\
S_w
\end{bmatrix}
\]  

(1)

\[V_{UN} + V_{VN} + V_{WN} = 0\]  

(2)

\[
\begin{bmatrix}
V_{UN} \\
V_{VN} \\
V_{WN}
\end{bmatrix}
= \frac{2}{3}V_{dc}
\begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
S_u \\
S_v \\
S_w
\end{bmatrix}
\]  

(3)

Applying the Clarke transformation to these output voltages, equation (4), it leads to a 2-D space vector \( V \) with the same instantaneous information in a stationary reference frame. The behavior of this vector is well known: when a balanced and sinusoidal three phase voltage system is analyzed, with constant magnitude \( V_0 \) and constant frequency \( \omega_0 \), the space voltage representation is a vector of constant length \( V_0 \) rotating with constant angular speed \( \omega_0 \). Likewise, when this transformation is applied to the inverter phase-to-neutral voltages the result are eight different vectors according to the eight possible switches states (see equation (5) and Fig. 2). The rotating reference vector \( v_o^* \) must be inside the hexagon for linear modulation.

\[
V = \begin{bmatrix}
V_\alpha \\
V_\beta
\end{bmatrix}
= \begin{bmatrix}
\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\
0 & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3}
\end{bmatrix}
\begin{bmatrix}
V_{UN} \\
V_{VN} \\
V_{WN}
\end{bmatrix}
\]  

(4)

Fig. 2: Space vectors of inverter voltages and other definitions.
\[ V_x = \frac{2}{3} V_{dc} e^{j(x-1)\pi/3}, \quad x = 1, 2, \ldots, 6 \]
\[ V_x = 0, \quad x = 0, 7 \]  
(5)

To obtain the required output space vector \( \mathbf{v}_o^* \), that usually is a rotating vector with constant module and pulsation, conduction times of the inverter switches are modulated according to the angle and magnitude of that reference. The null vectors (\( V_Z \equiv V_0 = V_7 \)), and two space vectors adjacent to \( \mathbf{v}_o^* \), \( V_i \) and \( V_j \), are chosen and modulated as follows:

\[ \mathbf{v}_o^* = d_z V_Z + d_i V_i + d_j V_j \]  
(6)

Duty cycles of these vectors can be computed using polar coordinates (see [1] for further details) by means of (7) [1], [6], [10], [11], where \( \theta_r \) is the relative angle between \( \mathbf{v}_o^* \) and \( V_i \) vectors (0 \( \leq \) \( \theta_r \) \( \leq \) \( \pi/3 \)), and \( m \) is the modulation index (0 \( \leq \) \( m \) \( \leq \) 1) defined as \( m = \sqrt{3 \cdot V_o/V_{dc}} \).

\[ d_i = m \sin(\frac{\pi}{3} - \theta_r) \]
\[ d_j = m \sin(\theta_r) \]
\[ d_z = 1 - d_i - d_j \]  
(7)

This method calculates all information needed to generate suitable switching sequences (usually \( d_0 = d_7 = d_Z / 2 \)), but its straightforward implementation needs the computation of two ‘\( \sin(x) \)’ functions [11] and one division by \( V_{dc} \). This division can be computed using a polynomial approach, but trigonometric functions require either big lookup tables or a time consuming process.

In addition, a previous operation is required: the sector of \( \mathbf{v}_o^* \) must be determined, because it is necessary to know which one of the inverter voltage vectors are adjacent to the given reference (see Fig. 2), in order to solve (7). For this operation the reference phase value can be used [12].

### A known improvement of SVM

The objective of space vector PWM technique is to approximate the reference phase-to-neutral voltage vector \( [v_{UN}^* \, v_{VN}^* \, v_{WN}^*]^T \) or its equivalent space vector \( \mathbf{v}_o^* \) by a combination of the eight switching patterns. One simple means of approximation is to require the average output of the inverter in a small switching period \( T_S \) to be the same as the average of \( \mathbf{v}_o^* \) in the same period, which for a switching frequency much higher than the fundamental one it is assumed constant during one switching cycle. This requirement, known as “per-carrier-cycle volt-second balance” principle [5], has been shown in (6), but can be exploited in a different way:

\[ T_Z \cdot V_Z + T_i \cdot V_i + T_j \cdot V_j = \int_{t}^{t+T_S} v_o^* \cdot dt \equiv v_o^* \cdot T_S \]  
(8)

As the value of \( T_Z \) has no effect in (8) because of the null magnitude of \( V_Z \), this equation can be used to compute \( T_i \) and \( T_j \) as shown in (9), where \( \mathbf{P} \) is defined at (10); \( T_Z \) is finally computed using (11).

\[
\begin{bmatrix}
T_i \\
T_j
\end{bmatrix} = \mathbf{P}^{-1} \cdot \begin{bmatrix}
v_{o\alpha}^* \\
v_{o\beta}^*
\end{bmatrix} \cdot T_S
\]
(9)

\[ \mathbf{P} \equiv \begin{bmatrix}
v_{o\alpha} & V_{ja} \\
v_{o\beta} & V_{jb}
\end{bmatrix} \]  
(10)

\[ T_Z = T_S - T_i - T_j \]  
(11)
The matrix $P$ is sector dependent and can be arranged (see table I) in such a way that a one-upper-switch-on vector ($V_1$, $V_3$ or $V_5$) is chosen as $V_i$ and a two-upper-switches-on vector ($V_2$, $V_4$ or $V_6$) is $V_j$. Hence it is easier for a digital signal processor (DSP) the generation of the typical $V_0 - V_i - V_j - V_0$ sequence during $T_S$, thus reducing the average switching frequency. These expressions have been widely used [8], [9], [12] rather than (7) because it is easy to implement them on DSP processors.

### Table I: Transformation matrices for each sector under SVM.

<table>
<thead>
<tr>
<th>Sector I</th>
<th>Sector II</th>
<th>Sector III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P = \begin{bmatrix} \frac{1}{\sqrt{3}} &amp; \frac{1}{\sqrt{3}} &amp; 0 \ \frac{1}{\sqrt{3}} &amp; \frac{1}{\sqrt{3}} &amp; 0 \end{bmatrix} V_{dc}$</td>
<td>$V_i = V_1$ and $V_j = V_2$</td>
<td>$P = \begin{bmatrix} \frac{1}{\sqrt{3}} &amp; 0 &amp; \frac{1}{\sqrt{3}} \ 0 &amp; \frac{1}{\sqrt{3}} &amp; \frac{1}{\sqrt{3}} \end{bmatrix} V_{dc}$</td>
</tr>
<tr>
<td>$V_i = V_3$ and $V_j = V_4$</td>
<td>$V_i = V_3$ and $V_j = V_5$</td>
<td>$V_i = V_4$ and $V_j = V_6$</td>
</tr>
</tbody>
</table>

### Proposed method for SVM

Computations involved in (9) and (11) are not complex and several implementations of that method are used in the reviewed literature. However, a simpler set of equations can be obtained solving them in an analytical way. To do so, it will be assumed, without loss of generality, a reference phase voltage between 0 and $\pi/3$, thus $V_1$ and $V_2$ will be used jointly with $V_0$ and $V_7$ to generate $v_{o*}$. Equation (9) and table I yield to:

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} V_{dc} & \frac{1}{\sqrt{3}} V_{dc} \\ 0 & \frac{1}{3} V_{dc} \end{bmatrix}^{-1} \begin{bmatrix} v_{o*} \\ v_{o*} \end{bmatrix} T_S$$

Equation (12) it leads to:

$$T_1 = \frac{\frac{1}{3} v_{o*}}{V_{dc}} - \frac{\sqrt{3} v_{o*}}{V_{dc}} T_S$$

$$T_2 = \frac{\sqrt{3} v_{o*}}{V_{dc}} T_S$$

However, applying (4) to (13) it yields to even simpler expressions (see [6] for a different approach):

$$T_1 = \frac{v_{UN}^* - v_{YN}^*}{V_{dc}} T_S$$

$$T_2 = \frac{v_{YN}^* - v_{WN}^*}{V_{dc}} T_S$$

In the same way, equating (11) it yields to the following expression, related with (14):

$$T_Z = \left(1 + \frac{v_{WN}^* - v_{UN}^*}{V_{dc}} \right) T_S$$
Equations (14) and (15), valid for sector I, show that SVM duty cycles can be computed without using the space vector transformation: actually, only the original reference voltages in the three-phase stationary reference frame are needed. The corresponding expressions for all sectors are shown in table II.

Table II: Proposed expressions to compute SVM duty cycles.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Duty cycles</th>
<th>Sector</th>
<th>Duty cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>I 0–π/3</td>
<td>( d_i = T_1 / T_S = (v_{UN}^* - v_{VN}^*) / V_{dc} )</td>
<td>IV π–4π/3</td>
<td>( d_i = T_3 / T_S = -(v_{VN}^* - v_{WN}^*) / V_{dc} )</td>
</tr>
<tr>
<td></td>
<td>( d_j = T_2 / T_S = (v_{YN}^* - v_{YW}^*) / V_{dc} )</td>
<td></td>
<td>( d_j = T_5 / T_S = -(v_{YN}^* - v_{UN}^*) / V_{dc} )</td>
</tr>
<tr>
<td>II π/3–2π/3</td>
<td>( d_i = T_3 / T_S = -(v_{UN}^* - v_{VN}^*) / V_{dc} )</td>
<td>V 4π/3–5π/3</td>
<td>( d_i = T_5 / T_S = -(v_{YN}^* - v_{UN}^*) / V_{dc} )</td>
</tr>
<tr>
<td></td>
<td>( d_j = T_2 / T_S = -(v_{YN}^* - v_{UN}^*) / V_{dc} )</td>
<td></td>
<td>( d_j = T_4 / T_S = -(v_{VN}^* - v_{WN}^*) / V_{dc} )</td>
</tr>
<tr>
<td>III 2π/3–π</td>
<td>( d_i = T_3 / T_S = (v_{VN}^* - v_{WN}^*) / V_{dc} )</td>
<td>VI 5π/3–2π</td>
<td>( d_i = T_5 / T_S = (v_{WN}^* - v_{UN}^*) / V_{dc} )</td>
</tr>
<tr>
<td></td>
<td>( d_j = T_2 / T_S = (v_{WN}^* - v_{UN}^*) / V_{dc} )</td>
<td></td>
<td>( d_j = T_6 / T_S = (v_{VN}^* - v_{WN}^*) / V_{dc} )</td>
</tr>
<tr>
<td></td>
<td>( d_Z = T_Z / T_S = 1 + (v_{WN}^* - v_{UN}^*) / V_{dc} )</td>
<td></td>
<td>( d_Z = T_Z / T_S = 1 + (v_{WN}^* - v_{UN}^*) / V_{dc} )</td>
</tr>
</tbody>
</table>

These SVM duty cycles may also be computed using the line-to-line representation of the reference voltages \( v_{UV}^* = v_{UN}^* - v_{VN}^* \); \( v_{VW}^* = v_{VN}^* - v_{WN}^* \); \( v_{WU}^* = v_{WN}^* - v_{UN}^* \) yielding to even simpler and faster expressions.

Moreover, the choice of the reference sector is also an easy task when line-to-line instantaneous voltages are used rather than space vector ones. For example, on sector I, it is straightforward to verify that \( v_{UN}^* \geq v_{VN}^* \geq v_{WN}^* \), thus \( v_{UV}^* \geq 0 \) and \( v_{VW}^* \geq 0 \) there. Similar expressions can be found for all sectors, as shown below.

Using all these ideas, the SVM timing computation can be completed using the following algorithm. Its implementation is straightforward using high level or even low level DSP programming languages:

Algorithm I: SVM implementation using the proposed method.

\[
\begin{align*}
k_{\text{sum}} &= T_S / V_{dc}; \\
\text{if } (v_{uv}^* > 0) \{ & \text{// continue from left) } \\
\text{if } (v_{vw}^* > 0) \{ & \text{// This is for sector I} \\
T_i &= k_{\text{sum}} * v_{uw}^*; \quad V_i = V_i (100); \\
T_j &= k_{\text{sum}} * v_{wu}^*; \quad V_j = V_j (110); \\
\} & \text{// This is for sector IV} \\
\text{if } (v_{vw}^* < 0) \{ & \text{// This is for sector V} \\
T_i &= -k_{\text{sum}} * v_{uw}^*; \quad V_i = V_i (001); \\
T_j &= -k_{\text{sum}} * v_{wu}^*; \quad V_j = V_j (011); \\
\} & \text{// This is for sector VI} \\
\text{else } \{ & \text{// This is for sector II} \\
T_i &= k_{\text{sum}} * v_{uw}^*; \quad V_i = V_i (001); \\
T_j &= k_{\text{sum}} * v_{wu}^*; \quad V_j = V_j (110); \\
\} & \text{// This is for sector III} \\
\text{else } \{ & \text{// This is for sector III} \\
T_i &= -k_{\text{sum}} * v_{uw}^*; \quad V_i = V_i (010); \\
T_j &= -k_{\text{sum}} * v_{wu}^*; \quad V_j = V_j (101); \\
\} & \text{// This is for all sectors} \\
T_Z &= T_S - T_i - T_j; \\
\end{align*}
\]

Equivalence between SVM and PWM

Equation (14) also produces an additional result, pointed out previously in the literature: SVM and PWM can be equivalent with small modifications [4], [9], [10]. Kwasinski et al. [9] state that SVM provides the same switch action and inverter output as a double-sided uniform-sampling PWM (UPWM) process with injected triple-\( n \) harmonics. SVM and UPWM always yield the same results for...
the active vectors, but SVM leaves a degree of freedom in the partitioning of zero vectors [5], [6], likewise PWM methods may add different zero-sequence signals to the modulation waves [5]. This paper agrees with that conclusion, but presents a different approach to this issue.

Firstly consider Fig. 3. It represents a classical PWM arrangement with distorted modulation, using $\pm V_{dc}/2$ as carrier limits: a triangular waveform with 25% magnitude (in bold dashed line) has been added to all arm voltage references to extend up the modulation index [2], [10]; as this signal only has triple-$n$ harmonics and they form zero-sequence systems, they produce no currents in floating loads. All voltages in this figure are referred to the DC bus midpoint, named 'M' in Fig. 1: carrier limits, phase voltage modulators, and also the AC neutral point voltage, $v_{NM}$, whose significant component matches the zero-sequence signal injected.

![Fig. 3: Classical PWM arrangement with distorted modulators.](image)

The same situation may be considered from a different point of view for symmetrical loads. When all voltages are referred to the AC neutral floating point main voltage, named 'N', Fig. 3 is transformed into Fig. 4. Phase-to-neutral voltage references have no zero-sequence component (their sum is zero), so they have no distortion; the voltage of the DC bus midpoint, named ‘M’, referred to the load neutral point, is also a triangular wave (at least its significant component) but now its sign is changed; finally, carrier limits are not constant, because of the variation of the $v_{MN}$ voltage main component. Noticeably, a digital implementation of this arrangement would be complicated, but it reveals interesting details.

![Fig. 4: A different perspective of PWM, referring all signals to the load neutral point.](image)

In this alternative PWM arrangement (Fig. 4), a high frequency carrier signal of constant height (equal to $V_{dc}$), but with time varying limits, intercepts three pure-sinusoidal phase-to-neutral voltage references and decides the switch states as usual ('0' when carrier is above each reference, '1' when it is below) [2]. Then, it follows that, for sector I (these results can be applied to all sectors), the time
period required by vector $V_1$ (100) is proportional to $T_S/V_{dc}$, an almost constant value, and also proportional to the difference between phase voltages $v_{UN}$ and $v_{VN}$ (matching what was stated in table II and algorithm I). In the same way, vector $V_2$ (110) must be proportional to the difference between phase voltages $v_{YN}$ and $v_{YN}$. Null vectors $V_0$ (000) and $V_7$ (111) fill the spare time at 50%. Actually, the same results are obtained when Fig. 3 is used instead of Fig. 4, because zero-sequence components modify all modulator waves in the same amount at any time.

A second detail clearly shown in Fig. 4 is the relationship between $v_{MN}$ and the partitioning of $T_Z$: active vectors ($V_i$ and $V_j$) are related only with the line-to-line output voltages [6], while zero vectors ($V_0$ and $V_7$) decide the shape of the zero-sequence voltage injected. A triangular wave results for $v_{MN}$ when $T_0$ equals $T_7$, but other options are clearly possible [5], [6].

**Simulations and experimental results**

Several simulations and physical implementations have been developed to validate these results. A 10 kVA 3-phase VSI connected to an L-R-E load (see the structure in Fig. 1) has been simulated using our own graphical environment [13]; its main results are shown in Figs. 5 and 6. In such simulation, the DC-link was $V_{dc} = 595$ V, the line-to-line 50 Hz output voltage was $V_{ac} = 400$ V (rms), the switching frequency was $f_S = 6$ kHz and the electrical load was resistive ($R = 16$ Ω) connected by means of a simple inductance ($L = 2000$ µH).

Inverter duty cycles were computed on every switching period using the equations shown in table II with $d_0 = d_7$, then PWM signals were generated using a triangular carrier with a resolution of 250 ns neglecting dead-band effects, and all simulated signals were sampled at 200 KSPS. The output voltage (Fig. 5-c) is so noisy because of the pure resistive nature of the load (i.e. $E = 0$ V).

Fig. 5: Simulation of a three-phase 10 kVA VSI connected to a symmetrical and pure resistive load. Four images are (a) inverter phase-to-neutral voltage for phase 'U', (b) inverter gate signals through a low-pass filter (second order at 1 kHz), (c) phase-to-neutral voltages on the resistive load, and (d) neutral point voltage, referred to the DC midpoint, through a low-pass filter (second order at 1 kHz).
In order to investigate the validity of these ideas and their practical capabilities, experimental results were obtained by applying the proposed SVM method using a Texas Instruments TMS320F2812 development board that achieves 150 MIPS. The source code was written in C and compiled using Code Composer Studio. It generates three 50 Hz sinusoidal references, selects the inverter sector and computes all switching duty cycles based on the proposed algorithm; then it configures three PWM generators to work at 5 kHz. The main results can be seen in Figs. 7 and 8.

Conclusion

This paper has presented a new, faster and, most important, simpler method to compute SVM duty cycles without using either trigonometric functions or even Park or Clarke transformations. The result is a light-weight algorithm easier to understand. Its implementation is straightforward in low-end digital signal processors or even small microcontrollers.

The relationship between SVM and PWM has also been explained using a different point of view: referring all voltages to the AC neutral point. Doing so, arm voltage references have no distortion, but time varying limits appear for PWM carrier. The advantage is that they allow an equally wide linear modulation range with no need of distorted modulation, and it also explains how triple-$n$ harmonics are injected with no effects on inverter output voltages when floating and symmetrical loads are used.

References


